1850MNRAS..10..182C

If $\mathfrak{B}_{\circ} = \frac{1}{a_3} \mathbf{B}_{\circ} \quad \mathfrak{B}_{1} = \frac{3\, a'}{a_4} \mathbf{B}_{1} \quad \mathfrak{B}_{2} = \frac{3 \cdot 5\, a'^2}{4\, a_5} \quad \mathbf{B}_{2}$, &c. and

If $\frac{a^{12}}{a^2} = x$, the expressions for $\mathbf{B}_0$, $\mathbf{B}_1$, $\mathbf{B}_2$ are of the form

$$1 + \alpha_1\, x + \alpha_2\, x^2 + \alpha_3\, x^3 + \&c.\,;$$

and the following table, calculated by Mr. Farley, shows the values of $A_1$, $A_2$, $A_3$, &c., $\alpha_1$, $\alpha_2$, $\alpha_3$, &c.; and it will be seen that the coefficients A *converge*, while, on the contrary, the coefficients $\alpha$ *diverge*.

| | $\mathbf{B}_0$ | $\mathbf{B}_{11}$ | $\mathbf{B}_{22}$ | | $\mathbf{B}_0$ | $\mathbf{B}_{11}$ | $\mathbf{B}_{22}$ |
|---|---|---|---|---|---|---|---|
| $A_1$ | 2·2500 | 1·5625 | 1·5326 | $\alpha_1$ | 2·2500 | 1·5625 | 1·5326 |
| $A_5$ | ·4007 | ·3461 | ·3277 | $\alpha_5$ | 7·3280 | 3·2319 | 2·9896 |
| $A_{10}$ | ·1992 | ·1838 | ·1731 | $\alpha_{10}$ | 13·6910 | 5·0339 | 4·4391 |
| $A_{15}$ | ·1330 | ·1263 | ·1192 | $\alpha_{15}$ | 20·0560 | 6·7571 | 5·7684 |
| $A_{20}$ | ·1010 | ·0964 | ·0919 | $\alpha_{20}$ | 26·4210 | 8·4490 | 7·0460 |

In computing an isolated case of $\mathfrak{B}$, it would not be worth while to procure the values of A; but in forming an extended table of the quantities $\mathfrak{B}$, they may be employed with advantage. The quantities A are also interesting, as affording the means of resolving any series into factors. See a recent memoir by M. Cauchy in the *Comptes Rendus*.

If the series

$$1 + \alpha_1\, x + \alpha_2\, x^2 + \alpha_3\, x^3 + \&c. = 1 + X,$$

$$\text{the logarithm} = X - \frac{X^2}{2} + \frac{X^3}{3} - \frac{X^4}{4} + \&c.\,;$$

and, by expanding, the expression for the logarithm might be obtained in terms of $\alpha_1$, $\alpha_2$, $\alpha_3$, &c.; but as the indices of the quantities increased, the law would not be easy to exhibit, and the equations would be altogether unmanageable for the computor.

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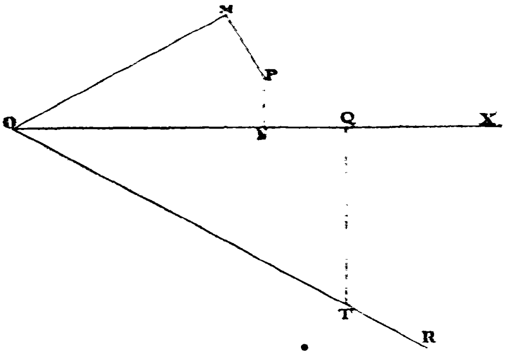
## *Description of an Instrument for calculating the Correction of a Transit Observation for Collimation, Level, and Azimuth Errors.* By Professor Challis.

"The collimation, level, and azimuth errors of a transit being respectively $a$, $b$, $c$, in seconds of space, the zenith distance of the observed object being $z$, and its north polar distance $\delta$, the instrument I am about to describe performs the calculation of $(a + b \cos z + c \sin z) \times \frac{\operatorname{cosec} \delta}{15}$, that is, the reduction of the observed time of transit to the time of meridian transit. The manner of

1850MNRAS..10..182C

effecting this depends on the following geometrical considerations. For simplicity, $a$, $b$, $c$, $z$, and $\delta$ will at first be supposed to be positive quantities. From an origin O draw a line of abscissæ O X, and another line O M, making with O X an angle equal to $z$.



Take O M equal in length to $b$ on an arbitrary scale, and from M draw M P at right angles to O M towards O X, and equal in length to $c$. Then, P N being drawn perpendicularly on O X, O N is the sum of the projections of O M and M P on O X, that is, $b \cos z + c \sin z$. Set off on O X the additional quantity N Q equal to $a$; then $\text{O Q} = a + b \cos z + c \sin z$. Draw an indefinite line O R, making with O X, on the side opposite to O M, an angle equal to $90^\circ - \delta$, so that the angle M O R is equal to the latitude of the Observatory. Lastly, draw QT at right angles to O X, cutting O R in T. Then, if O R be made a scale of equal parts commencing at O, and if the unit of this scale be fifteen times the unit of the scale from which $a$, $b$, and $c$ were cut off, it is clear that the reading O T will be $\text{O Q} \times \frac{\operatorname{cosec} \delta}{15}$, which is the quantity it was required to calculate.

"The preceding geometrical considerations apply in the mechanical construction of the instrument as follows. The point O is the centre of a brass circular plate, movable about a vertical axis passing through this point; and on the plate are engraved, first, the scale line O R, continued both ways from O across the plate, for the purpose of taking account of negative corrections, and next, two scale lines at right angles to each other, in both directions from O, to serve for setting off positive and negative level and azimuth errors, the positive direction of the scale line for level error making, with the positive direction of the correction-scale O R, an angle equal to the latitude of the Observatory. At a short distance from the circular plate, and in a direction parallel to the arbitrary fixed direction O X, there is placed a straight edge, for the purpose of guiding the motion of two bars, which carry two fine parallel threads of blackened unspun silk over the surface of the plate, the direction of the threads being perpendicular to the straight edge. The interval between the threads is made equal to the collimation error (without regarding its sign), by means of a scale engraved on a brass plate to which one of the bars is at-

1850MNRAS..10..182C

tached, and an index fixed to the brass plate to which the other bar is attached, the two plates being clamped together by a screw, when the threads are set at the required interval. The screw-head serves for a handle to move the threads. For the purpose of setting the circle in any proposed instance, its upper surface is graduated close to the circumference from 0° to 130° of positive north polar distance, and from 0° to 60° of negative north polar distance, the place of the zero of the graduation being chosen arbitrarily. The fixed index which points to the north polar distance is in such a position that its reading is zero, when the scale O R coincides in direction with the threads and the positive side of the scale is directed *from* the straight edge which guides the motion of the threads. Also the direction in which the positive numbers of the graduation increase, is determined by making the reading +90°, when the positive direction of O R coincides with the positive direction of O X.

" According to these arrangements the calculation is performed simply as follows. First, the point P is marked on the circular plate by means of the level and azimuth scales, and the threads are separated by an interval equal to the collimation error by means of the collimation scale. These operations, once performed, apply to all observations for which the collimation, level, and azimuth errors are the same. Next, for a particular observation, the circle is moved till the index of the graduation points to the north polar distance of the object: then, the point P being bisected by one of the threads, the other cuts off from O R the distance O T, which being read by the scale is the quantity sought. The bisection of P is made by one or the other of the threads, according as the collimation error is positive or negative.

" It is proper to mention several other details in the construction of the machine, and certain precautions for ensuring expedition and accuracy in the use of it. For marking readily the position of the point P, a ruled square, the sides of which serve for the level and azimuth scales, is engraved on the plate, its centre coinciding with the centre of motion of the circle. The side of the square is 6 inches, and half an inch corresponds to one second of space, so that the extent of the scales is ±6″. The ruled lines are one-twentieth of an inch apart, and, consequently, mark tenths of a second. By estimation the position of P may be marked to the hundredth of a second. The diameter of the circle is 15 inches, and the greatest amount of correction that can be read from the correction scale is $\pm 1^s$. The hundredths of a second of time are marked on this scale, and thousandths may be read by estimation. The collimation-scale extends to 7″. The two threads are adjustible to parallelism, and when the index of the collimation-scale points to zero, it is so arranged that one thread shall be vertically above the other, the threads being let into grooves at their extremities. The lower of the two pieces of brass to which the bars carrying the threads are attached, is fastened to a rectangular piece of box-wood, which slides in a brass trough, one vertical side of which

1850MNRAS..10..182C

is the guiding edge for the common motion of the threads, and against the other a spring presses to keep the box-wood close to the guiding surface. The trough rests on three screws working in the mahogany support of the instrument, and is adjustible to position by means of these screws. The words "Collimation," "Level," and "Azimuth," are engraved to distinguish the scales, and the directions in which the co-ordinates of P are to be set off are indicated by the signs + and —. Also the figures of the correction-scale have the signs + or — attached to them. The bars carrying the threads have the letters A and B engraved on them, and a card stuck in a bracket indicates which of the two threads is to be used for the bisection of P, by reference to one or the other of these letters, according as the collimation-error is positive or negative. A black-lead pencil will mark the place of P with sufficient distinctness on the brass plate by two short intersecting lines, and the lines can be readily effaced. The above precautions make it almost impossible to commit any mistake in the use of the instrument.

"It remains to mention the advantages which the instrument offers, and the degree of accuracy of which it is susceptible. For calculating the reduction to meridian transit, it has been usual to form a table of the coefficients of the collimation, level, and azimuth errors, the argument of which is the north polar distance, whence the coefficient for a proposed north polar distance may be readily deduced for insertion in the transit calculation-book. This being done, the calculation consists of three multiplications and three additions for each object. For the sake of saving time the multiplications are performed by a sliding scale. To give an idea of the advantage gained by the new instrument, it may be stated that it saves the trouble of inserting the coefficients, and the operation for obtaining the whole correction occupies no more time than one multiplication by the sliding scale. The common method may be inaccurate to one hundredth, or even two hundredths of a second, if the errors of the multiplications all lie in the same direction, whilst the machine by moderate care will give the nearest hundredth of a second. It is, however, to be said, that the indications of the instrument become more uncertain in proportion as the north polar distances are less, on account of the small inclination of the threads to the correction-scale, and that in the above construction this scale is too short to allow of reading off for north polar distance less than $30^\circ$, when the level and azimuth errors are greater than $5''$ and of opposite signs, and the collimation-error is greater than $2''$. To meet the case of small north polar distances an additional scale is engraved on the upper edge of that side of the trough which guides the threads, the unit being a second of space. The zero of the scale coincides with the position of a thread when it bisects the centre of the circle, and the positive numbers increase in the positive direction of O X. The process for reading off from this scale is precisely the same as for the correction-scale, but the quantity read off is $a + b \cos z + c \sin z$,

1850MNRAS..10..186C

which has to be multiplied by the coefficient for collimation-error. Even this use of the instrument saves considerable time, and for stars near the pole, as *Polaris*, I find it to be somewhat more accurate than the ordinary method.

"The construction of the instrument, which required no little nicety of workmanship, has been executed for me by Mr. Simms in a very satisfactory manner.

"If the instrument be used in a latitude different from that for which it was constructed, a slight additional calculation will be required for determining the position of the point P, which may be performed by the instrument itself. For if $l$ be the colatitude of the observatory for which the instrument was made, and $z$ the zenith distance of an object, and if $l'$, $z'$ be the corresponding arcs for another observatory, we have,

$$a + b \cos z' + c \sin z' = a + b' \cos z + c' \sin z,$$
$$\text{if } b' = c \sin (l - l') + b \cos (l - l')$$
$$\text{and } c' = c \cos (l - l') - b \sin (l - l')$$

Hence $b'$ and $c'$ are the co-ordinates of P to be used at the latter place."

## *Description of an Instrument for obtaining the Corrections to be applied to Transit Observations.** By Mr. R. C. Carrington, of the Observatory, Durham.

Mr. Carrington, having heard that Professor Challis had proposed an instrument for correcting transit observations, set himself to work to solve the same problem. Finding his construction different from that of Professor Challis, he has had his instrument executed by Messrs. Troughton and Simms, and presented it to the Astronomical Society.

Expanding the usual formula, viz.

$$\frac{c}{\sin \delta} + \frac{i \cos (\delta - l)}{\sin \delta} + \frac{a \sin (\delta - l)}{\sin \delta},$$

($c$, $i$, and $a$ being the collimation, inclination, and azimuth corrections in units of 15″,)

Mr. Carrington arrives at the equivalent formula,

$c'' + c' \operatorname{cotan} \delta + c \operatorname{cosec} \delta$; where $c$ remains as before, and

$c' = i \cos l - a \sin l$, and $c'' = i \sin l + a \cos l$.

The geometrical construction on which Mr. Carrington's instrument depends, is this.

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* In May last, the President particularly called the attention of the meeting to Mr. Carrington's instrument, and to Mr. Johnson's beautiful and accurate annual volume of observations.